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Switch off the mobile phone
Identify all sheets with your name and
Solve each problem in diferente sheets
Justify all answers
Duration: 2h30m
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Problem 1 ( 9,5 )
Consider the reinforced concrete structure represented in the figure. It is constituted by two adjacent buildings separated by an expansion joint 3 cm long and filled with neoprene, that works like a strut. The beams are rigid and the columns axially undeformable.

a) Evaluate the mass and stiffness matrices considering the degrees of freedom $q_{1}$ and $q_{2}$.
b) Evaluate the periods and vibration modes using the characteristic equation.

For the purpose of a linear dynamic analysis by response spectrum acording to EC 8 and the portuguese National Annex, consider that the structure is of Class of Importance III, it is located in Portimão (zone 1.1) in soil type $C$ and it is acted upon by an earthquake type 1 (zone 1.1). Consider a behavior factor $q=3$.
c) Evaluate the shear force on the columns of structure B and the axial force on the expansion joint. (4,0)
d) Assuming the stiffness of the expansion joint is zero, check if the buildings pound against each other $(1,5)$

If you didn't answer question $b$ ), consider $T_{1}=0,7 \mathrm{~s}, \mathrm{~T}_{2}=0,4 \mathrm{~s}, v_{1}=\left\{\begin{array}{l}1 \\ 1\end{array}\right\}$ e $v_{2}=\left\{\begin{array}{l}-0,5 \\ 1\end{array}\right\}$
Problem $2(5,0)$
Consider the single degree of freedom system indicated in the figure. The beams are rigid and the columns axially undeformable. $E I_{p}=25000 \mathrm{kNm}^{2}, \mathrm{~m}=10$ ton/m e $\xi=5 \%$
a) Evaluate the period of the structure.
b) Calculate the maximum amplitude of vibration in the $1^{\text {st }}$ floor if the structure is acted upon in the $2^{\text {nd }}$ floor by forces equal to $F=100 \cos 8 t$ $[k N]$ and $F=200 \cos 4 t[k N]$. If you didn't answer to the previous question, consider $\mathrm{T}=0,7 \mathrm{~s}$
c) Comment the relations between the values of the forces and displacements calculated in the answer to question b).


## Problem $3(5,5)$

a) The following figure schematically presents several mixed frame-wall structures. Put the structures in order from the worse to the better from the point of view of seismic conception. Justify your answer. $(1,0)$

(a)

(b)

(c)

(d)
b) In the application of Capacity Design principles to mixed frame-wall structures, what is the advange of allowing the formation of plastic hinges only at the base? Justify your answer.
c) Explain what is a response spectrum. Draw qualitatively the elastic response spectrum for earthquakes type 1 and 2 (on the same figure), justifying the adopted representation.
d) Define return period. Explain why a temporary structure does not need to be designed for the same seismic action as current structures. Of what factors depends the level of design seismic action? What is the minimum number of stations necessary to determine an epicenter? Justify your answer describing the process of determination of epicentres.
e) Consider the continuos model represented in figure below. For the purpose of determination of the fundamental frequency by the Rayleigh method indicate what are the cinematic and static boundary conditions to consider for the shape function $\Psi(x)$.


Quadro NA.I - Aceleração máxima de referência $a_{\mathrm{RR}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ nas várias zonas sísmicas

| Acção sísmica Tipo 1 |  | Acção sísmica Tipo 2 |  |
| :---: | :---: | :---: | :---: |
| Zona Sísmica | $a_{\mathrm{gR}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Zona Sísmica | $a_{\mathrm{RR}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| 1.1 | 2,5 | 2.1 | 2,5 |
| 1.2 | 2,0 | 2.2 | 2,0 |
| 1.3 | 1,5 | 2.3 | 1,7 |
| 1.4 | 1,0 | 2.4 | 1,1 |
| 1.5 | 0,6 | 2.5 | 0,8 |
| 1.6 | 0,35 | - | - |

f) NA-3.2.2.2(2)P

Em Portugal, para a definição dos espectros de resposta elásticos o valor do parâmetro $S$ deve ser determinado através de:
para $a_{\mathrm{g}} \leq 1 \mathrm{~m} / \mathrm{s}^{2} \quad S=S_{\max }$
para $1 \mathrm{~m} / \mathrm{s}^{2}<a_{\mathrm{g}}<4 \mathrm{~m} / \mathrm{s}^{2} \quad S=S_{\max }-\frac{S_{\max }-1}{3}\left(a_{\mathrm{g}}-1\right)$
para $a_{\mathrm{g}} \geq 4 \mathrm{~m} / \mathrm{s}^{2} \quad S=1,0$
em que:
$a_{\mathrm{g}} \quad$ valor de cálculo da aceleração à superficie de um terrleno do tipo $\mathrm{A}, \mathrm{em} \mathrm{m} / \mathrm{s}^{2}$;
$S_{\max } \quad$ parâmetro cujo valor é indicado nos Quadros NA-3.2 e NA-3.3.
Em Portugal, para a definição dos espectros de resposta elásticos para a Acção sísmica Tipo 1 devem adoptar-se os valores do Quadro NA-3.2 em vez do Quadro 3.2.

Quadro NA-3.2 - Valores dos parâmetros definidores do espectro de resposta elástico para a
Acção sísmica Tipo 1

| Tipo de <br> Terreno | $S_{\max }$ | $T_{\mathrm{B}}(\mathrm{s})$ | $T_{\mathrm{C}}(\mathrm{s})$ | $T_{\mathrm{D}}(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: |
| A | 1,0 | 0,1 | 0,6 | 2,0 |
| B | 1,35 | 0,1 | 0,6 | 2,0 |
| C | 1,6 | 0,1 | 0,6 | 2,0 |
| D | 2,0 | 0,1 | 0,8 | 2,0 |
| E | 1,8 | 0,1 | 0,6 | 2,0 |

h) NA-4.2.5(5)P

Em Portugal, os coeficientes de importância a adoptar são os indicados no Quadro NA.II
Quadro NA.II - Coeficientes de importância $\gamma$

| Classe de <br> Importância | Acção sísmica <br> Tipo 1 | Acção sísmica Tipo 2 |  |
| :---: | :---: | :---: | :---: |
|  |  | Continente | Açores |
| I | 0,65 | 0,75 | 0,85 |
| II | 1,00 | 1,00 | 1,00 |
| III | 1,45 | 1,25 | 1,15 |
| IV | 1,95 | 1,50 | 1,35 |

(4)P Para as componentes horizontais da acção sísmica, o espectro de cálculo, $S_{\mathrm{d}}(T)$, é definido pelas seguintes expressões:

$$
\begin{gather*}
0 \leq T \leq T_{\mathrm{B}}: S_{\mathrm{d}}(T)=a_{\mathrm{g}} \cdot S \cdot\left[\frac{2}{3}+\frac{T}{T_{\mathrm{B}}} \cdot\left(\frac{2,5}{q}-\frac{2}{3}\right)\right]  \tag{3.13}\\
T_{\mathrm{B}} \leq T \leq T_{\mathrm{C}}: S_{\mathrm{d}}(T)=a_{\mathrm{g}} \cdot S \cdot \frac{2,5}{q}  \tag{3.14}\\
T_{\mathrm{C}} \leq T \leq T_{\mathrm{D}}: S_{\mathrm{d}}(T)\left\{\begin{array}{l}
=a_{\mathrm{g}} \cdot S \cdot \frac{2,5}{q} \cdot\left[\frac{T_{\mathrm{C}}}{T}\right] \\
\geq \beta \cdot a_{\mathrm{g}}
\end{array}\right.  \tag{3.15}\\
T_{\mathrm{D}} \leq T: \quad S_{\mathrm{d}}(T)\left\{\begin{array}{l}
=a_{\mathrm{g}} \cdot S \cdot \frac{2,5}{q} \cdot\left[\frac{T_{\mathrm{C}} T_{\mathrm{D}}}{T^{2}}\right] \\
\geq \beta \cdot a_{\mathrm{g}}
\end{array}\right. \tag{3.16}
\end{gather*}
$$

Excertos da NP EN 1998-1 (Anexo Nacional NA, 2009)

$$
a_{g}=a_{g R} \gamma_{1} \quad=0,2
$$

$\left|K-p^{2} M\right|=0 \quad D=F M \quad D V=\frac{1}{p^{2}} V$
$V_{i}^{\top} M V_{j}= \begin{cases}0 & i \neq j \\ M_{j} & i=j\end{cases}$
$V_{i}^{\top} K V_{j}=\left\{\begin{array}{cc}0 & i \neq j \\ M_{j} p_{j}^{2} & i=j\end{array}\right.$
$\phi_{i}=\frac{V_{i}}{\sqrt{V_{i}{ }^{\top} M V_{i}}} \quad \bar{P}_{i x}=\phi_{i}^{\top} M 1_{x}$

|  |  | ${ }^{\text {a }}$ | $\mathrm{mmIIII}_{\mathrm{L}}{ }^{\text {a }}$ | $\stackrel{a}{a_{\mathrm{L}}^{\mathrm{z}^{\mathrm{o}}}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\left\\|\left\\|_{L}\right\\|^{\text {b }}\right.$ | abL | $\frac{1}{2} \mathrm{abL}$ | $\frac{1}{2} \mathrm{abL}$ | $\frac{1}{3} \mathrm{abL}$ |
|  |  | $\frac{1}{3} \mathrm{abL}$ | $\frac{1}{6} \mathrm{abL}$ | $\frac{1}{4} \mathrm{abL}$ |
|  |  |  | $\frac{1}{12} \mathrm{abL}$ | $\frac{1}{5} \mathrm{abL}$ |

$\ddot{\mathrm{q}}_{\mathrm{i} \alpha}^{\max }=\overline{\mathrm{P}}_{\mathrm{i} \alpha} \mathrm{S}_{\mathrm{a} i \alpha} \phi_{\mathrm{i}} \quad \quad \mathrm{q}_{\mathrm{i} \alpha}^{\max }=\overline{\mathrm{P}}_{\mathrm{i} \alpha} \mathrm{S}_{\mathrm{di} \mathrm{\alpha}} \phi_{\mathrm{l}}$
$\mathrm{S}_{\mathrm{dj}}=\frac{\mathrm{S}_{a j}}{4 \pi^{2} \mathrm{f}_{\mathrm{j}}^{2}} \mathrm{r}^{\max }=\sqrt{\sum_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{j}}^{\max }\right)^{2}}$
$\mathrm{p}^{2}=\mathrm{g} \frac{\int_{0}^{\ell} \mathrm{m}(x) q_{G}(x) d x+\sum_{\mathrm{i}} M_{i} q_{G}\left(x_{i}\right)}{\int_{0}^{\ell} \mathrm{m}(x)\left[q_{G}(x)\right]^{2} d x+\sum_{\mathrm{j}} M_{\mathrm{j}}\left[q_{G}\left(x_{j}\right]^{2}\right.}$


$\mathrm{p}^{2}=\frac{\int_{0}^{\ell} \operatorname{EI}(x)\left[\psi^{\prime \prime}(x)\right]^{2} d x+\sum_{\mathrm{i}} \mathrm{K}_{\Delta i}\left[\psi\left(x_{i}\right)\right]^{2}+\sum_{\mathrm{j}} \mathrm{K}_{\theta \mathrm{jj}}\left[\psi^{\prime}\left(x_{\mathrm{j}}\right)\right]^{2}}{\int_{0}^{l} \mathrm{~m}(x)[\psi(x)]^{2} d x+\sum_{\mathrm{m}} \mathrm{M}_{\mathrm{m}}\left[\psi\left(x_{m}\right)\right]^{2}+\sum_{\mathrm{n}} \mathrm{I}_{\theta n}\left[\psi^{\prime}\left(x_{n}\right)\right]^{2}}$
$\beta_{1}=\frac{1}{\sqrt{\left(1-\bar{\omega}^{2}\right)^{2}+(2 \zeta \bar{\omega})^{2}}}$

$$
\bar{\omega}=\frac{\omega}{\mathrm{p}}
$$


$\beta_{2}=\sqrt{1+(2 \zeta \pi)^{2}} \times \beta_{1}$

$$
\beta_{3}=\beta_{1} \times \sigma^{2}
$$

$$
p_{d}=p \sqrt{1-\zeta^{2}}
$$

$\left.\mathrm{q}(t)=\mathrm{e}^{-\zeta \rho t}\left(\mathrm{q}_{0} \cos \left(p_{d} t\right)\right)+\frac{\dot{\mathrm{q}}_{0}+\zeta \mathrm{p} \mathrm{q}_{0}}{\mathrm{p}_{\mathrm{d}}} \operatorname{sen}\left(p_{d} t\right)\right)+\frac{\mathrm{e}^{-\zeta \rho t}}{\mathrm{M} p_{\mathrm{d}}} \int_{0}^{\mathrm{t}} \mathrm{e}^{\zeta \rho \tau} \mathrm{Q}(\tau) \operatorname{sen}\left(p_{d}(t-\tau)\right) d \tau$
$q(t)=\beta_{1} \frac{Q}{k} \cos (\omega t-\phi) \quad \phi=\operatorname{arctg}\left(\frac{2 \xi \varpi}{1-\varpi^{2}}\right)$
$\mathrm{SRSS} \Rightarrow \frac{f_{i}}{f_{i-1}}>1,1$

